An Introduction to Overset Grids

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Stanford California, 2011

Top 3 reasons for using overset grids.

1. Complex geometry and accuracy: You need to solve a partial differential equation (PDE) on a complex geometry and require accurate representations at boundaries (e.g. boundary layers).
2. Moving geometry : overset grids provide fast moving grid generation and high quality grids.
3. Efficiency: overset grids can take advantage of fast and memory efficient algorithms for structured (and Cartesian) grids.
   - Example: 3D, 4th-order Maxwell : Cartesian grids are 25$\times$ faster than curvilinear grids which themselves are $2-10??\times$ faster than unstructured grids.
   - Example: multigrid solvers for overset grids: can be an order of magnitude faster (e.g. 50$\times$) and more memory efficient (e.g. 10$\times$) than the best Krylov based solvers.

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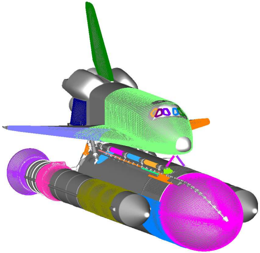
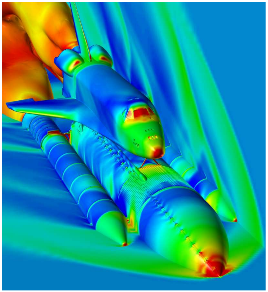
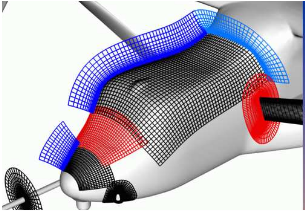
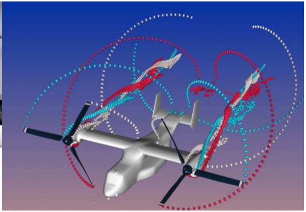
Overset grids are used to solve some of the most difficult CFD problems in aerospace.

Space shuttle figures courtesy of William Chan and Reynaldo Gomez.
V-22 Osprey figures courtesy of William Chan, Andrew Wissink and Robert Meakin.

Composite/ Chimeral/ Overset/ Overlapping Grids

A Short History

- Volkov, circa [1966] developed a *Composite Mesh* method for Laplace's equation on regions with piece-wise smooth boundaries separated by corners. Polar grids are fitted at corners to handle potential singularities.
- Starius, circa [1977] (student of H.-O. Kreiss) considered *Composite Mesh* methods for elliptic and hyperbolic problems – introduces a hyperbolic grid generator.
- Steger, circa [1980] independently conceives the idea of the overlapping grid, subsequently named the *Chimera* approach after the mythical Chimera beast having a human face, a lion's mane and legs, a goat's body, and dragon's tail. NASA groups develop grid generator PEGSUS, hyperbolic grid generation and flow solver Overflow (Steger, Benek, Suhs, Buning, Chan, Meakin, et. al.)
- B. Kreiss circa [1980] develops overlapping grid generator which subsequently leads to the CMPGRD grid generator [1983] (Chesshire, Henshaw) later leading to the Overture set of tools [1994].

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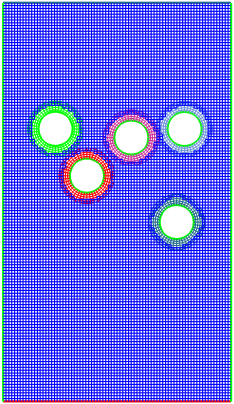
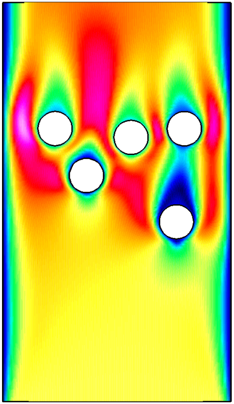
What are overlapping grids and why are they useful?

Overlapping grid: a set of structured grids that overlap.





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- High quality grids under large displacements.
- Cartesian grids for efficiency.
- Efficient for high-order accurate methods.

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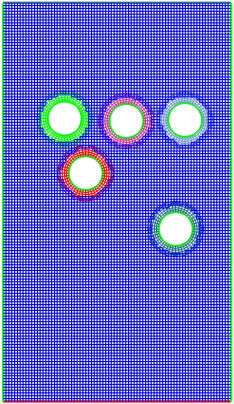

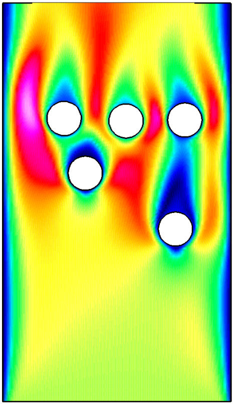

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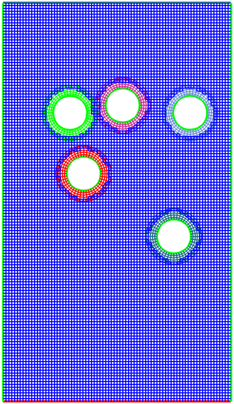

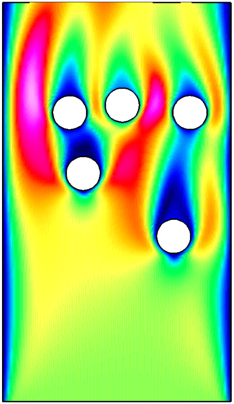

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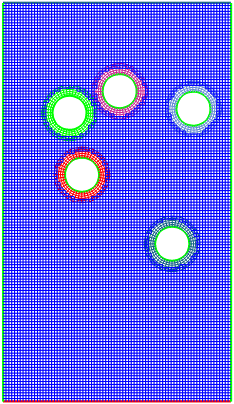

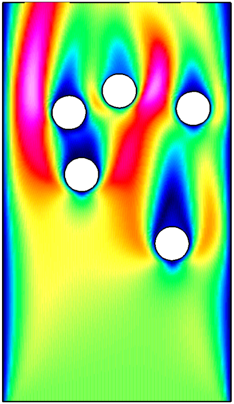

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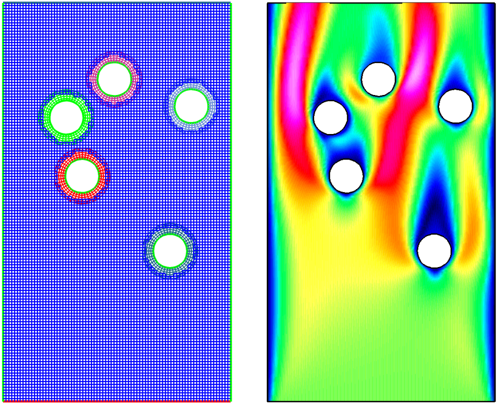

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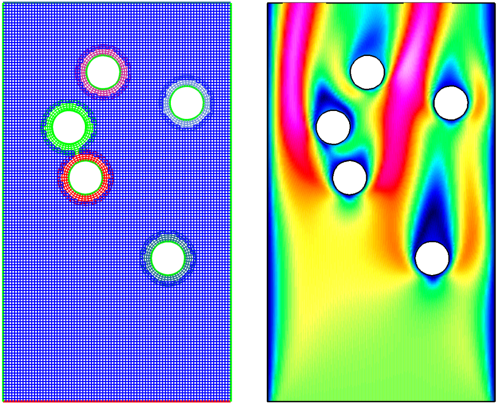

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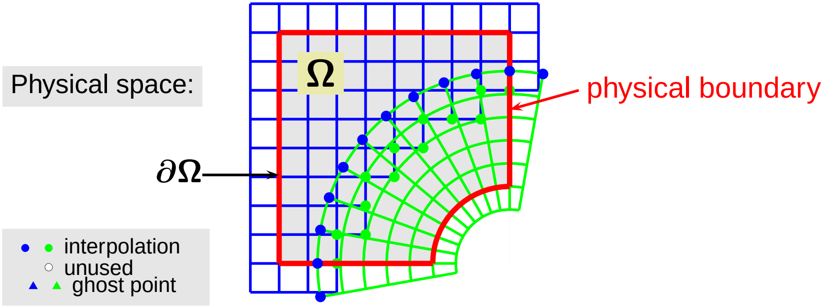
Physical space:
Ω
∂Ω
physical boundary
interpolation
unused
ghost point

Components of an Overlapping Grid

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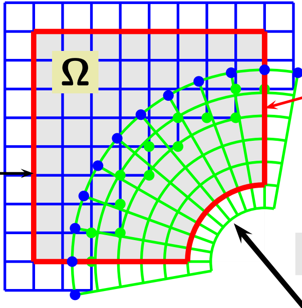

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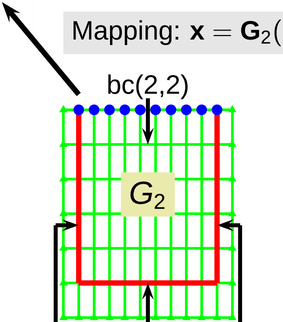
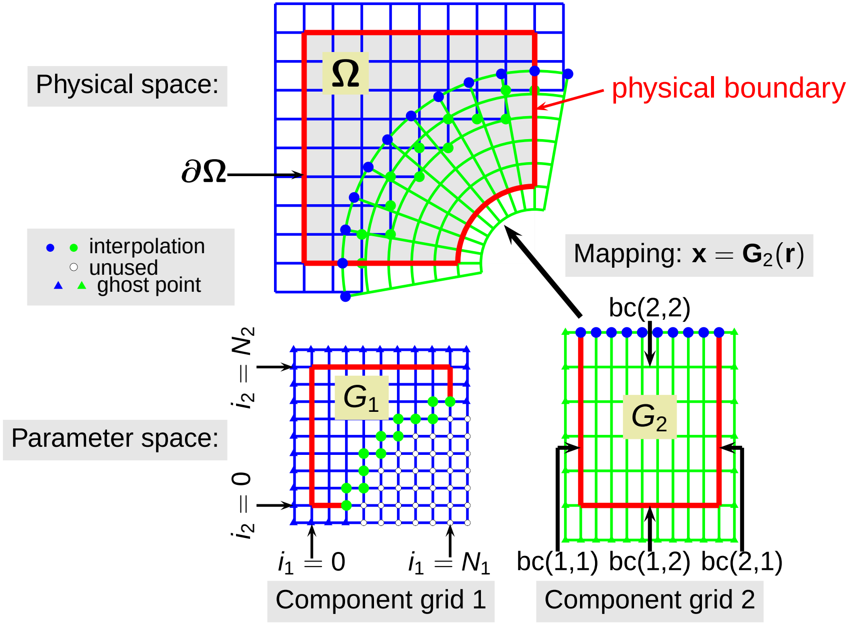
Mapping: $\mathbf{x} = \mathbf{G}_2(\mathbf{r})$

Parameter space:

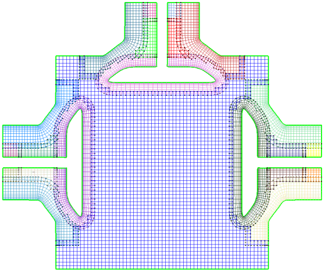


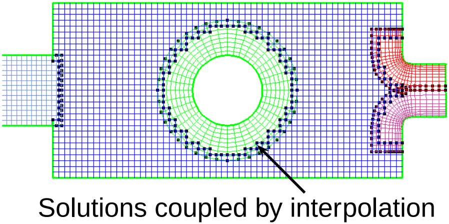
Component grid 2

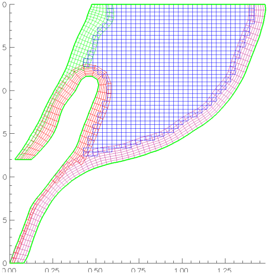
Components of an Overlapping Grid



Sample 2D overlapping grids (built with Ogen)







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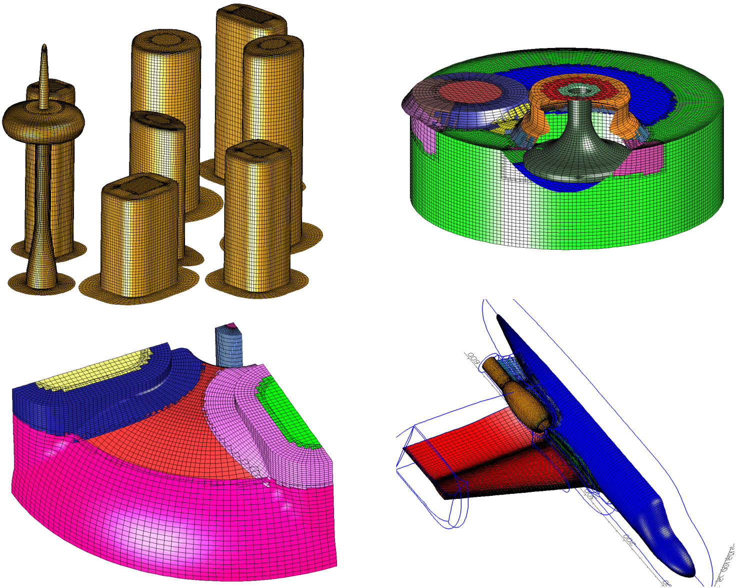

A one-dimensional overlapping grid example

To solve the advection-diffusion equation

$$
\begin{aligned}
u_t + a u_x &= \nu u_{xx}, && x \in (0,1) \\
u(0,t) = g_0(t), \quad u_x(1,t) &= g_1(t), && \text{(boundary conditions)} \\
u(x,0) &= u_0(x), && \text{(initial conditions)}
\end{aligned}
$$

introduce grid points on the two overlapping component grids,

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\begin{aligned}
x_i^{(1)} &= x_a + i\Delta x_1, & i &= -1,0,1,\ldots,N_1+1, & \Delta x_1 &= (x_d - x_a)/N_1 \\
x_j^{(2)} &= x_c + (j+1)\Delta x_2, & j &= -1,0,1,\ldots,N_2+1, & \Delta x_2 &= (x_b - x_c)/N_2
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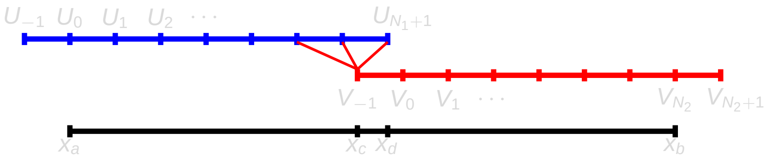
and approximations $U_i^n \approx u(x_i^{(1)}, n\Delta t)$, $V_i^n \approx u(x_i^{(2)}, n\Delta t)$.



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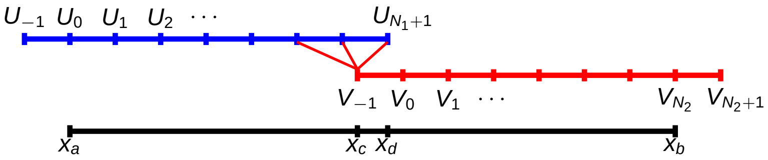

Given the solution at time t^n, compute the solution at time t^{n+1}:

$$(U_i^{n+1} - U_i^n)/\Delta t = -a\frac{U_{i+1}^n - U_{i-1}^n}{2\Delta x} + \nu\frac{U_{i+1}^n - 2U_i^n + U_{i-1}^n}{\Delta x^2}, \qquad i = 1, 2, \dots, N_1$$

$$(V_j^{n+1} - V_j^n)/\Delta t = -a\frac{V_{i+1}^n - V_{i-1}^n}{2\Delta x} + \nu\frac{V_{i+1}^n - 2V_i^n + V_{i-1}^n}{\Delta x^2}, \qquad j = 0, 2, \dots, N_2$$

$$U_0^{n+1} = g(t^n), \quad D_0 V_{N_2}^{n+1} = g_1(t^{n+1}), \qquad \text{(boundary conditions)}$$

$$U_{N_1+1}^{n+1} = (1-\alpha)(1-\frac{\alpha}{2})\, V_{-1}^{n+1} + \alpha(2-\alpha)\, V_0^{n+1} + \frac{\alpha}{2}(\alpha-1)\, V_1^{n+1}, \quad \text{(interpolation)}$$

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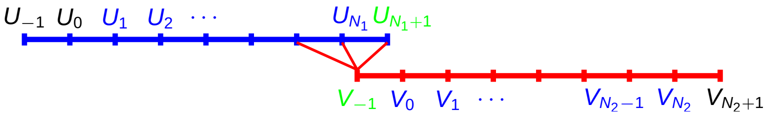

How to advance the solution on an overlapping grid.

(1) interior equations, (2) boundary conditions, (3) interpolation points.

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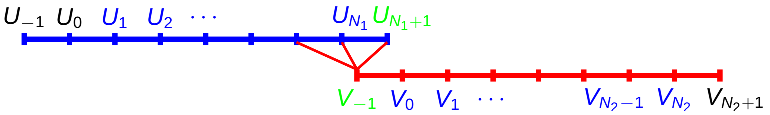

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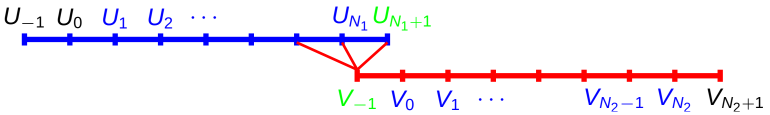


Theory for finite difference schemes

There is extensive numerical analysis theory underpinning this work.

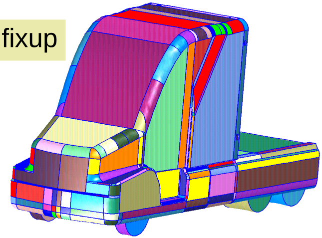
- classic von Neumann stability analysis (periodic domains).
- energy estimates (L_2-norm estimates).
- normal mode analysis, GKS theory (initial boundary value problems).

Some references:

• Gustafsson, Kreiss, Oliger, *Time Dependent Problems and Difference Methods*, (book).
• Strikwerda, *Finite Difference Schemes and Partial Differential Equations*, (book).
• Gustafsson, Kreiss, Sundström, *Stability Theory of Difference Approx. for Mixed Initial Boundary Value Problems, I. and II.*, Math. Comp.
• Starius, *On Composite Mesh Difference Methods for Hyperbolic Differential Equations*, Numer. Math.

From CAD to Mesh to Solution

1. Cad fixup



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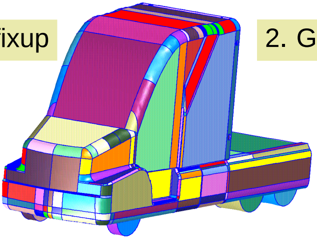
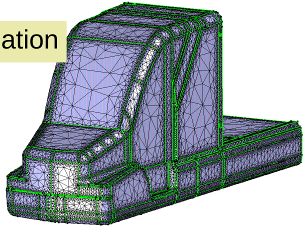


2. Global triangulation



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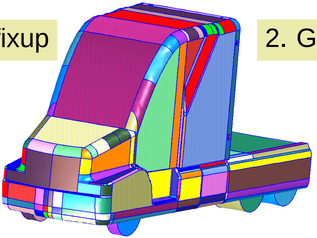

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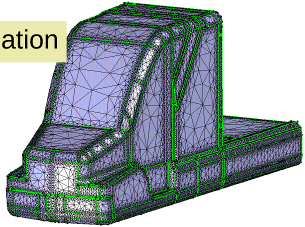
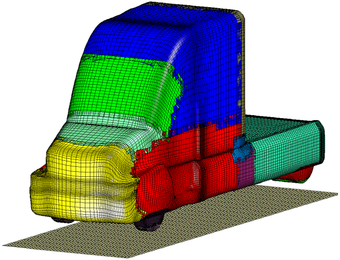


3. Overlapping grid



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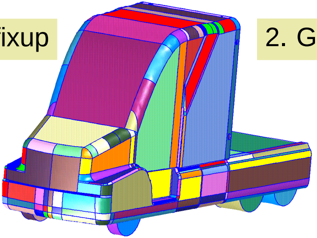
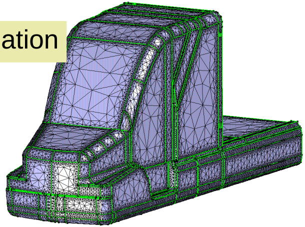
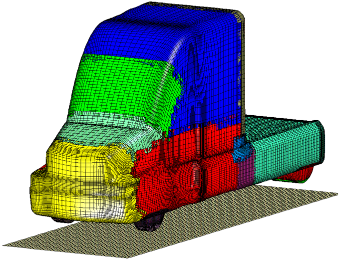


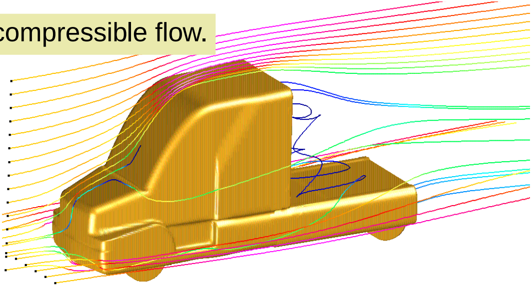
2. Global triangulation



3. Overlapping grid



4. Incompressible flow.



Software for overlapping grids.

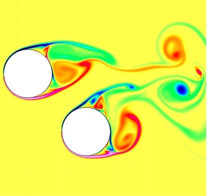
Here is a partial list of software for both grid generation and solving PDEs on overlapping grids (availability varies, e.g. some is export controlled).

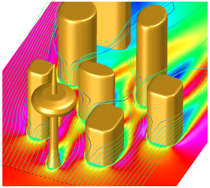
- Chimera Grid Tools : utilities, libraries and scripts for grid manipulation, component grid generation, and solution analysis.
- PEGSUS: (versions 4 and 5) : grid connectivity.
- SUGGAR, DIRTLIB : grid connectivity and interpolation utilities.
- Compressible Navier-Stokes solvers: OVERFLOW, BEGGAR, HELIOS, ADPDIS3D, SAFARI, ...
- Incompressible Navier-Stokes solvers: INS2D, INS3D, EllipSys3d, CFDShip-Iowa, ...
- Overture: grid generation and PDE solvers for fluid flows (Cgcns, Cgins), electromagnetics (Cgmx), elastic wave-equation (Cgsm), conjugate heat transfer and fluid structure interactions (Cgmp).

Applications and Movies

(taken from Overture based solvers)

Cgins: incompressible Navier-Stokes solver.



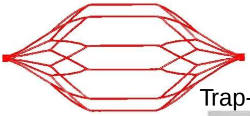


- 2nd-order and 4th-order accurate (DNS).
- support for moving rigid-bodies (not parallel yet).
- heat transfer (Boussinesq approximation).
- semi-implicit (time accurate), pseudo steady-state (efficient line solver), full implicit.

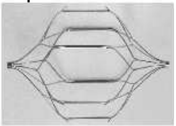
• WDH., *A Fourth-Order Accurate Method for the Incompressible Navier-Stokes Equations on Overlapping Grids*, J. Comput. Phys, **113**, no. 1, (1994) 13–25.

Flow past a blood-clot filter using cgins

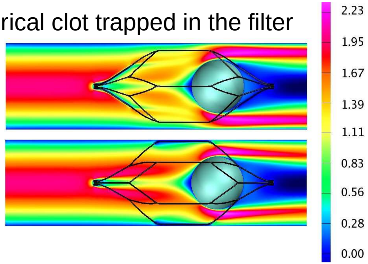
Overlapping grid for the filter



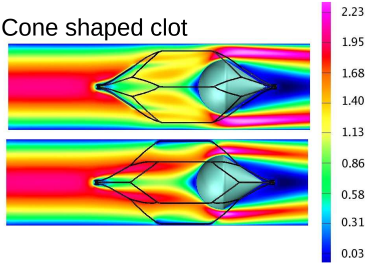
Trap-ease wire filter



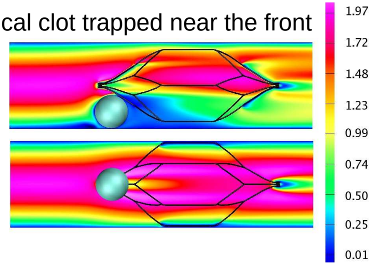
Spherical clot trapped in the filter



Cone shaped clot

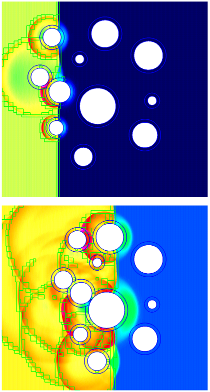


Spherical clot trapped near the front



M.A. Singer, WDH, S.L. Wang, *Computational Modeling of Blood Flow in the Trapease Inferior Vena Cava Filter*, Journal of Vascular and Interventional Radiology, **20**, 2009.

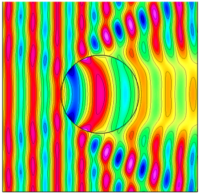
Cgcns: compressible N-S and reactive-Euler.

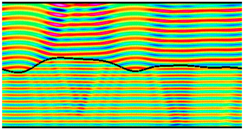


- reactive and non-reactive Euler equations, Don Schwendeman (RPI).
- compressible Navier-Stokes.
- multi-fluid formulation, Jeff Banks (LLNL).
- adaptive mesh refinement and moving grids.

- WDH., D. W. Schwendeman, *Parallel Computation of Three-Dimensional Flows using Overlapping Grids with Adaptive Mesh Refinement*, J. Comp. Phys. **227** (2008).
- WDH., DWS, *Moving Overlapping Grids with Adaptive Mesh Refinement for High-Speed Reactive and Nonreactive Flow*, J. Comp. Phys. **216** (2005).
- WDH., DWS, An adaptive numerical scheme for high-speed reactive flow on overlapping grids, J. Comp. Phys. **191** (2003).

Cgmx: electromagnetics solver.





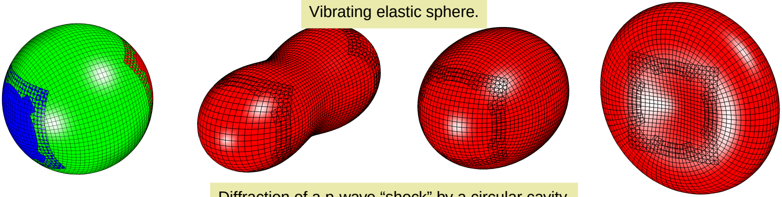
- fourth-order accurate, 2D, 3D.
- Efficient time-stepping with the modified-equation approach
- High-order accurate symmetric difference approximations.
- High-order-accurate *centered* boundary and interface conditions.

• WDH., *A High-Order Accurate Parallel Solver for Maxwell's Equations on Overlapping Grids*, SIAM J. Scientific Computing, **28**, no. 5, (2006).

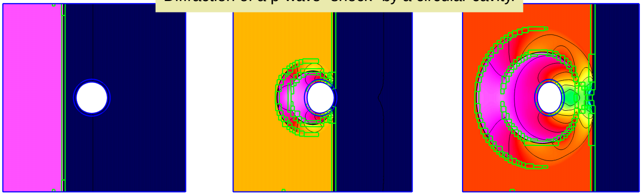
Cgsm: solve the elastic wave equation.

- linear elasticity on overlapping grids, with adaptive mesh refinement,
- conservative finite difference scheme for the second-order system,
- upwind Godunov scheme for the first-order-system.

Vibrating elastic sphere.



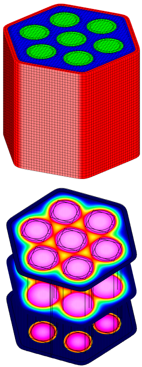
Diffraction of a p-wave “shock” by a circular cavity.



- D. Appelö, J.W. Banks , WDH, D.W. Schwendeman, *Numerical Methods for Solid Mechanics on Overlapping Grids: Linear Elasticity*, LLNL-JRNL-422223, submitted.

Cgmp: a multi-domain multi-physics solver.

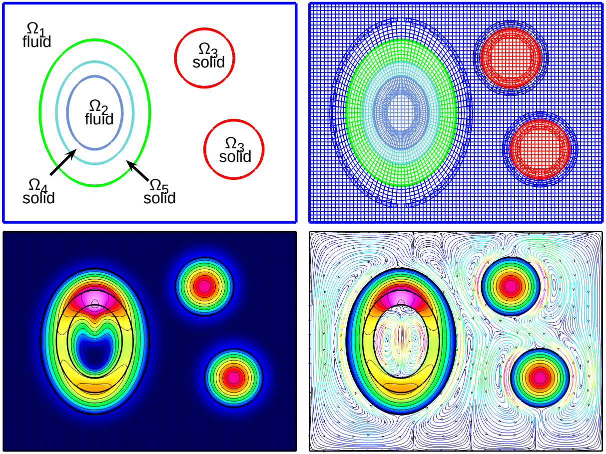
Conjugate heat transfer: coupling incompressible flow to heat conduction in solids.



- overlapping grids for each fluid or solid domain,
- a partitioned solution algorithm (separate physics solvers in each sub-domain),
- (cgins) incompressible Navier-Stokes equations (with Boussinesq approximation) for fluid domains,
- (cgad) heat equation for solid domains,
- a key issue is interface coupling.

• WDH., K. K. Chand, *A Composite Grid Solver for Conjugate Heat Transfer in Fluid-Structure Systems*, J. Comput. Phys, 2009.

The multi-domain composite grid approach



The fluid and solid sub-domains, overlapping grids and solution (temperature and streamlines) to a CHT problem. Solvers: cgins (fluid sub-domains), cgad (solid sub-domains), cgmp (coupled problem).

Summary: Overset Grids.

- Overset grids can be used to efficiently and accurately solve partial differential equations in complex geometry.
- Overset grids can be an order of magnitude faster and more memory efficient that unstructured grid algorithms.
- Overset grids are especially useful for problems with moving geometry.
- Automating the grid generation process is an important area of research.